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## A Magnetohydrodynamic Flow Angle Indicator

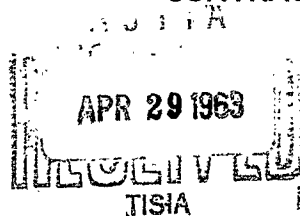
13 NOVEMBER 1962

*Prepared by A. E. FUHS and O. L. GIBB  
Physical Research Laboratory*

*Prepared for*  
COMMANDER HEADQUARTERS, BALLISTIC SYSTEMS DIVISION  
AIR FORCE SYSTEMS COMMAND  
UNITED STATES AIR FORCE  
*Norton Air Force Base, California*



LABORATORIES DIVISION • AEROSPACE CORPORATION  
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### ABSTRACT

By using the directional sensitivity of the electrical conductivity instrument, it is possible to measure the direction of the flow of a conducting gas. If only one coil set is used, a complex calibration procedure is necessary; however, by using two coil sets, the flow angle becomes a function only of the ratio of two signals. It is shown theoretically, and verified experimentally, that the signal varies as the cosine of the flow angle. The flow angle can be expressed analytically as a function of signal ratio. Calculated and observed signal ratios are in agreement. The precision of measurement is  $\pm (1 \text{ deg} + 5\%)$  for flow angles of  $+45$  to  $-45$  degrees.

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## I. INTRODUCTION

Information obtained from the flight of a magnetohydrodynamic (MHD) electrical conductivity meter aboard a reentry vehicle indicated that a correlation exists between signal amplitude and vehicle oscillations.<sup>1</sup> This correlation suggested the use of an electrical conductivity instrument as a local flow angle indicator and, with suitable calibration, as an angle-of-attack instrument for hypersonic vehicles. This application of the instrument would provide information on local flow angles which is pertinent to the study of hypersonic flow fields surrounding bodies at angle of attack.

The following sections of this report describe two different geometrical arrangements for an MHD flow angle indicator.

## II. DEPENDENCE OF SIGNAL ON FLOW ANGLE RELATIVE TO THE TRANSDUCER

The operation of the MHD electrical conductivity instrument is described in References 2 and 3. Briefly, operation is based on the ability of the instrument to detect induced currents which are established by the interaction of an applied magnetic field with a moving conductor, such as the ionized air of a plasma sheath, rocket exhaust, or arc plasma jet. The induced magnetic field is detected and measured by a sensing coil. Voltage at the terminals of the sensing coil,  $e$ , is expressed by<sup>2</sup>

$$e = - \frac{NA\mu_o}{4\pi} \frac{\partial}{\partial t} \int_V \frac{\sigma(\vec{U} \times \vec{B}) \times \vec{r}}{r^3} \cdot \vec{n} dV \quad (1)$$

where  $A$  represents the cross-sectional area of the sensing coil;  $N$ , the number of turns; and  $\vec{n}$ , the spatial orientation. The velocity of the conductor

is  $\vec{U}$ . The selected coordinate system is illustrated in Figure 1. Flow velocity is assumed to be

$$\vec{U} = \vec{e}_x U \cos \beta + \vec{e}_z U \sin \beta \quad (2)$$

Performing the vector manipulations of Equation (1), using the velocity given by Equation (2), the result for an E-lamination geometry ( $\vec{n} = \vec{e}_y$ ) is shown to be

$$e = - \frac{NA\mu_0}{4\pi} \frac{\partial}{\partial t} \int_V \frac{UxB_y \cos \beta + UzB_y \sin \beta}{r^3} dV \quad (3)$$

The term in  $\sin \beta$  is dropped because  $B_y$  is an odd function in  $x$ , and the integration over  $x$  has symmetrical limits.

For the pancake-coil geometry ( $\vec{n} = \vec{e}_x$ ), the sensing coil voltage is

$$e = - \frac{NA\mu_0}{4\pi} \frac{\partial}{\partial t} \int_V \frac{UzB_x \sin \beta - UzB_z \cos \beta - UyB_y \cos \beta}{r^3} dV \quad (4)$$

Once again, the term in  $\sin \beta$  integrates to zero due to symmetry.

With either geometrical arrangement, the signal reduces to

$$e = e_0 \cos \beta \quad (5)$$

where  $e_0$ , as determined by  $\sigma$  and  $U$ , is the signal for  $\beta = 0$ . To use one coil set to measure  $\beta$ , it would be necessary to know both  $e$  and  $e_0$ . However, if either two coil sets (as shown in Figure 2) or two sensing coils (Figure 3) were used, the dependence on  $\sigma$  and  $U$  would be removed.

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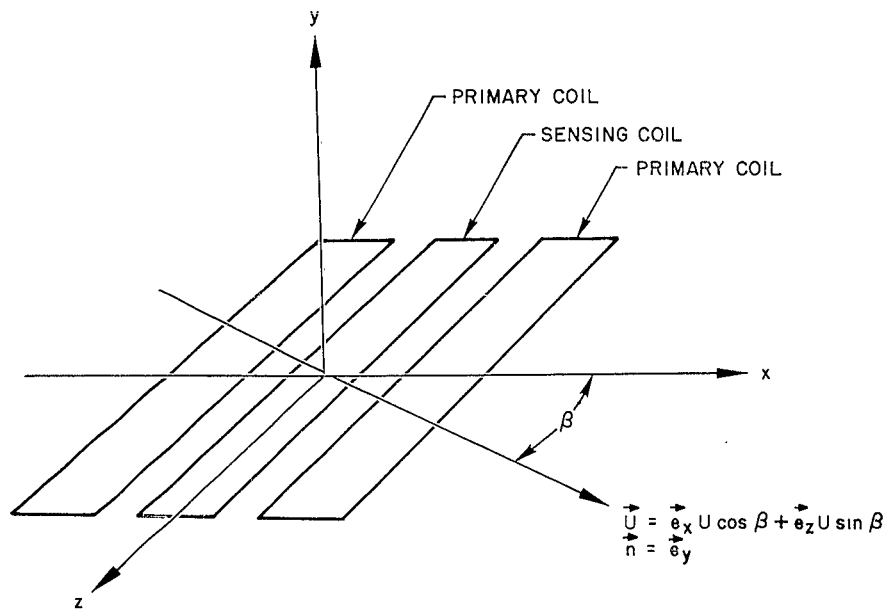
$$e = - \frac{NA\mu_0}{4\pi} \frac{\partial}{\partial t} \int_V \frac{Uz B_x \sin \beta - Uz B_z \cos \beta - Uy B_y \cos \beta}{r^3} dV \quad (4)$$

Once again, the term in  $\sin \beta$  integrates to zero due to symmetry.

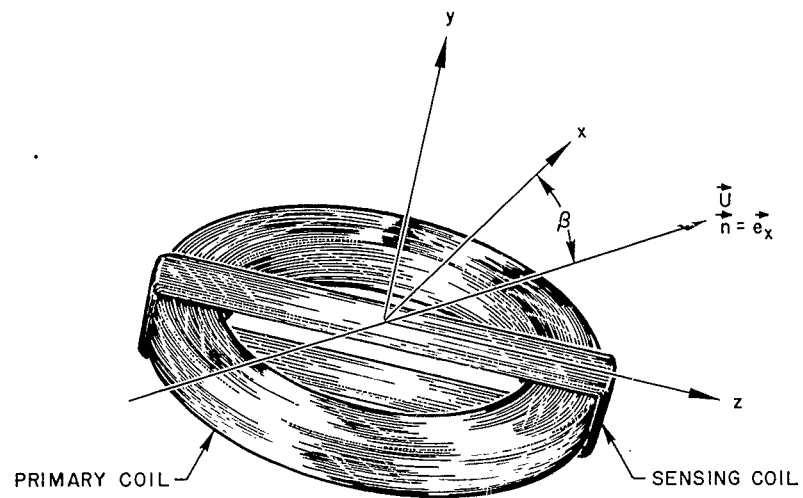
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E-Lamination Geometry



Pancake Coil Geometry

Figure 1. Geometry of the E-Lamination and Pancake Coil Arrangements



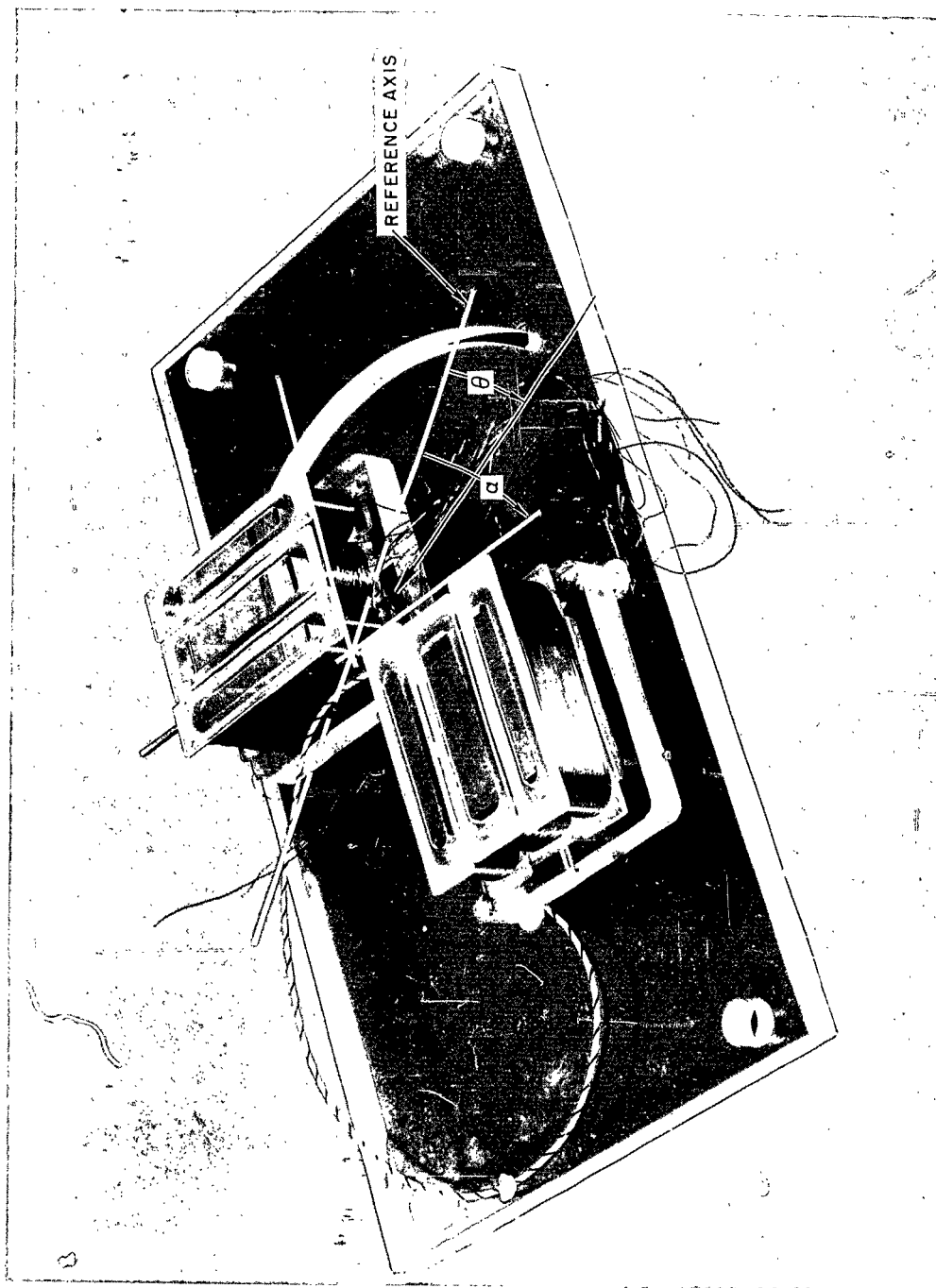


Figure 2. MHD Flow Angle Indicator Using E-Lamination Geometry

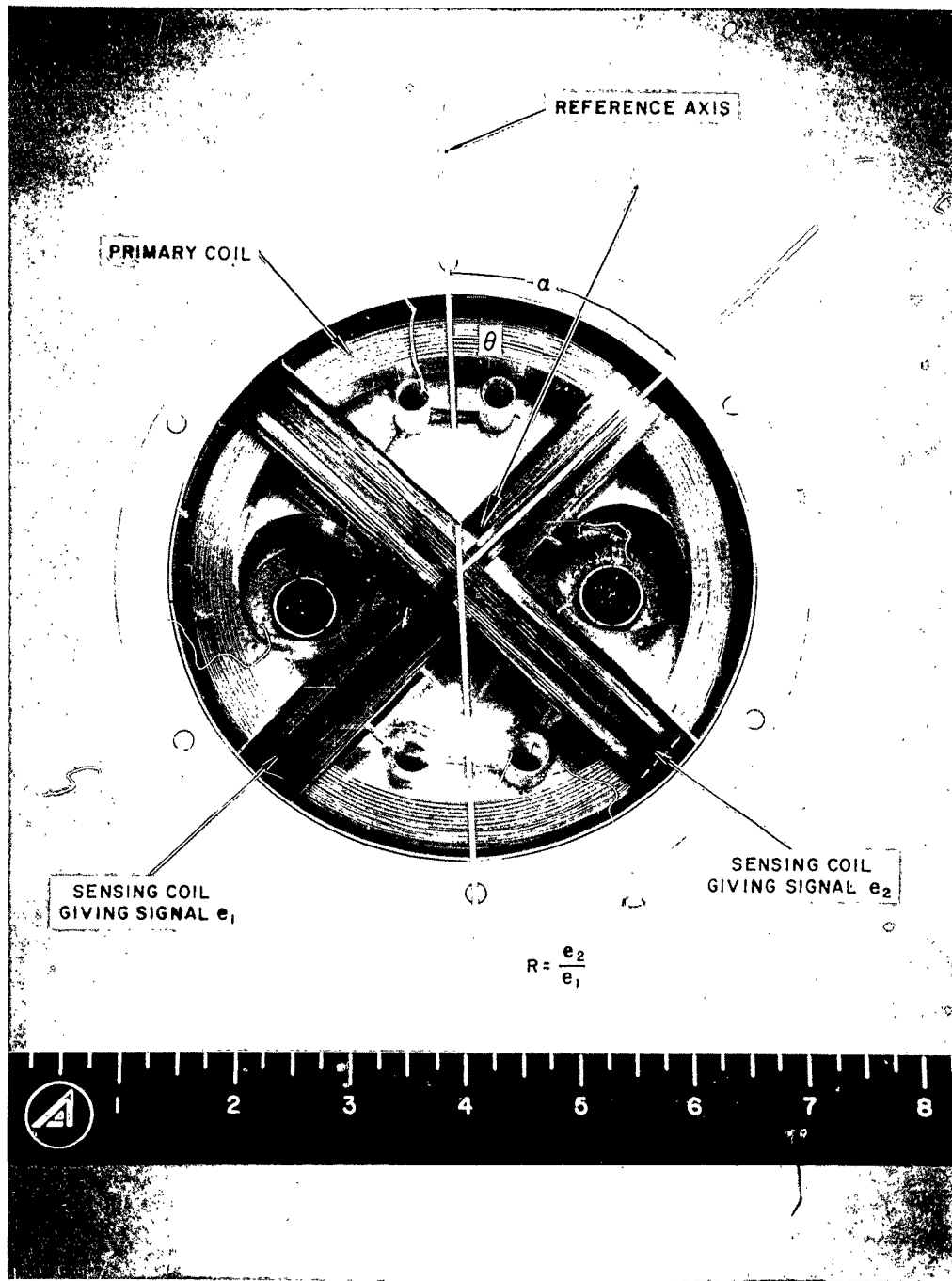


Figure 3. Crossed Sensing Coil Geometry

The angle of the coil set relative to a reference axis is represented by  $\alpha$ . (Refer to Figures 2 and 3.) The flow relative to the same reference axis is denoted by  $\theta$ . Positive  $\theta$  indicates an increase in the signal in sensing coil 1 and a decrease in the signal in sensing coil 2. The emf from sensing coil 1 is

$$e_1 = e_{10} \cos (\alpha - \theta) \quad , \quad (6)$$

and from sensing coil 2 is

$$e_2 = e_{20} \cos (\alpha + \theta) \quad . \quad (7)$$

Defining  $R$  as the ratio of signals  $e_2/e_1$ , Equations (6) and (7) can be solved for  $\theta$  in terms of  $R$  and  $\alpha$ . The result is<sup>4</sup>

$$\theta = \arctan \left( \frac{1 - R}{1 + R} \frac{1}{\tan \alpha} \right) \quad (8)$$

Calculations have been made for  $\alpha = 10, 20, 30, 40$ , and  $45$  degrees using Equation (8). Graphs of  $\theta$  versus  $R$  are presented in Figure 4 (for  $R \leq 1$ ) and Figure 5 (for  $R \geq 1$ ).

In the absence of plasma flow, a voltage develops in the sensing coil; this unwanted, but, nevertheless, ever-present voltage is termed the "null signal." The null signal can be held at a low level relative to signals due to plasma flow, but it cannot be eliminated. With the null signal included, the sensing coil voltage\* is

$$e = e_0 \cos (\alpha \pm \theta) + n \quad , \quad (9)$$

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\* Synchronous detection (see Reference 5) or phase sensitive demodulation is used so that voltages add algebraically, rather than vectorially.

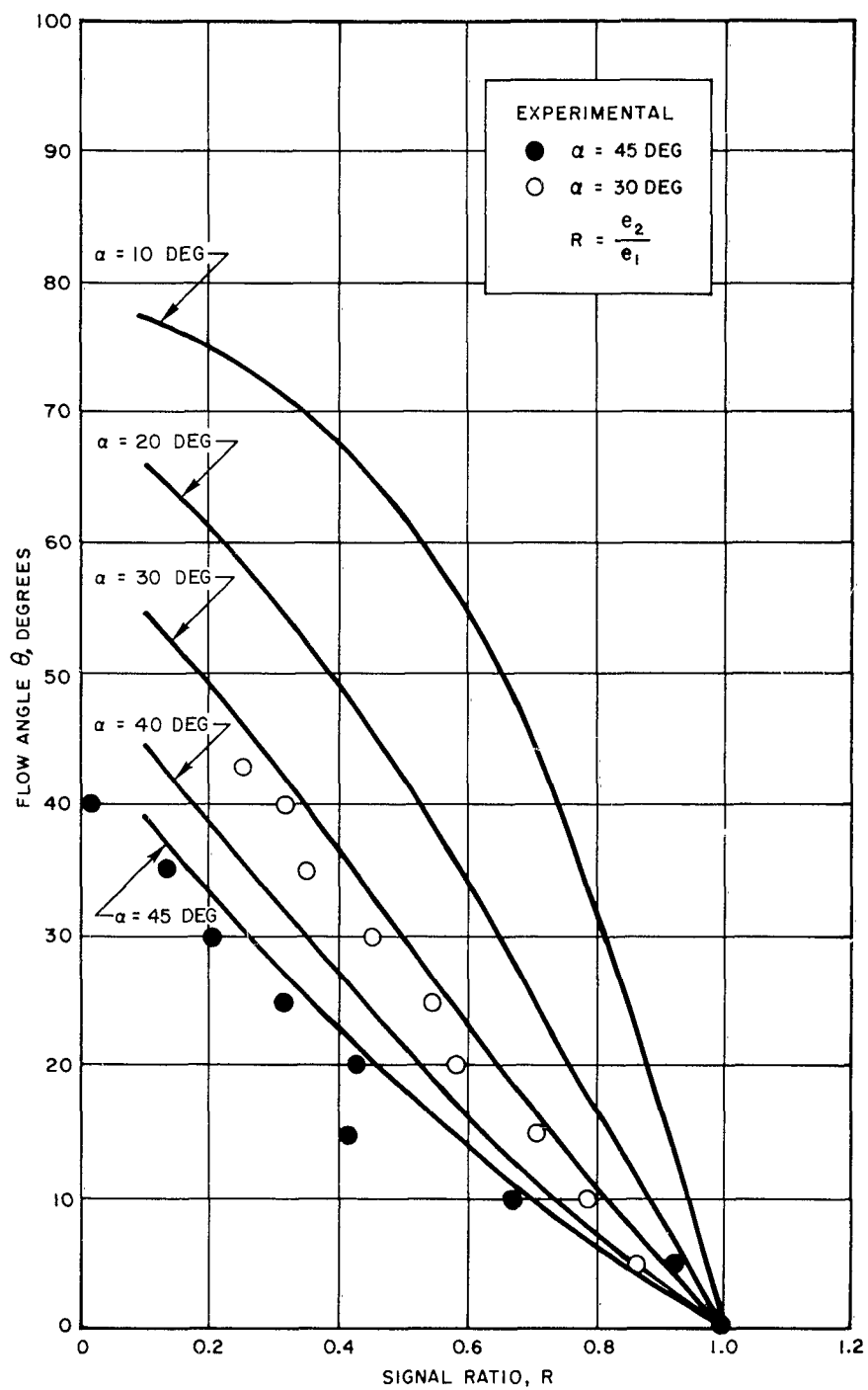


Figure 4. Calculated Curves of Flow Angle (Positive  $\theta$ ) as a Function of Signal Ratio, R, and Experimental Data Points

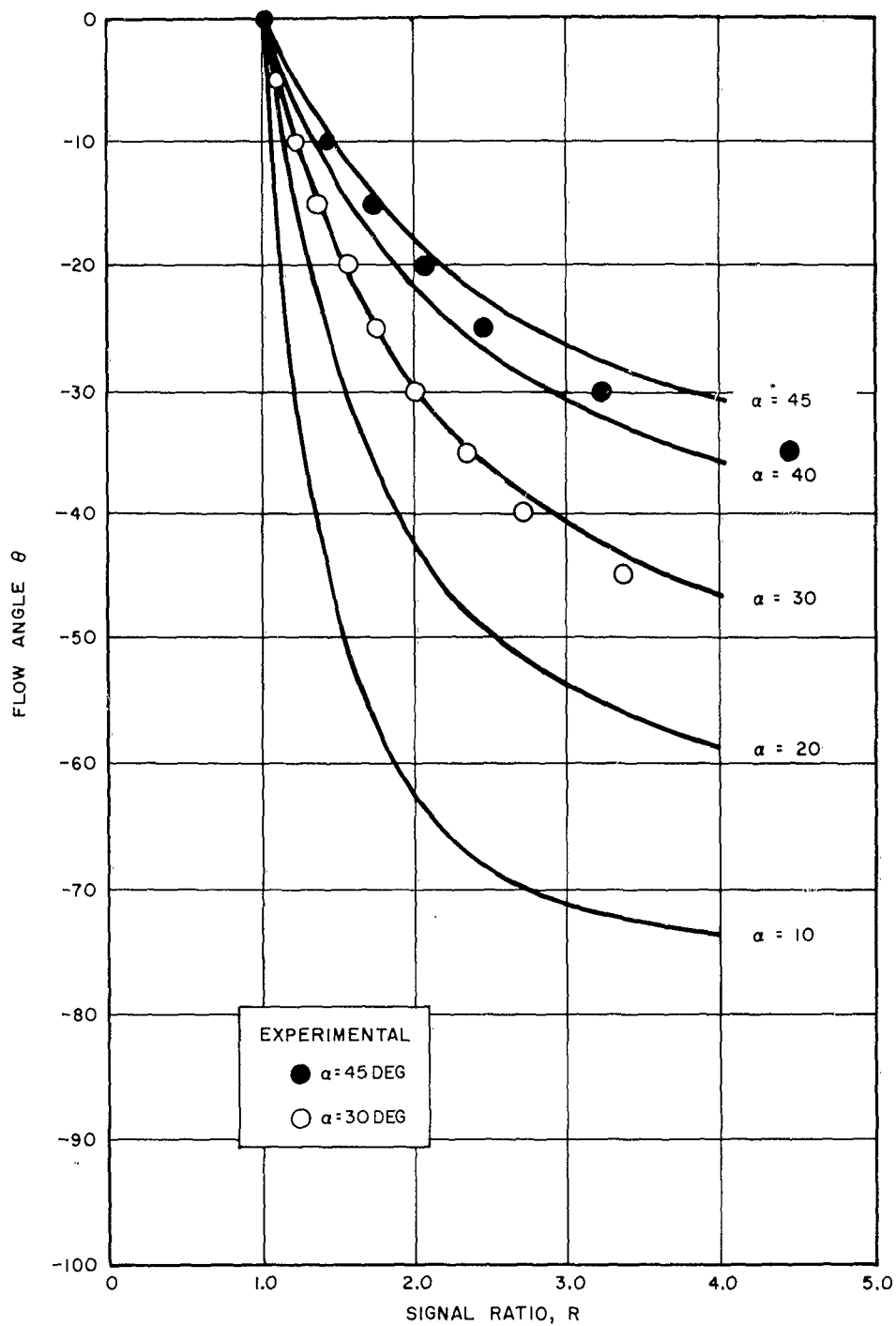


Figure 5. Calculated Curves of Flow Angle (Negative  $\theta$ ) as a Function of Signal Ratio,  $R$ , and Experimental Data Points

and the signal ratio is

$$R = \frac{\cos (\alpha + \theta) + (n_2/e_0)}{\cos (\alpha - \theta) + (n_1/e_0)} \quad (10)$$

It is obvious from Equation (10) that the sensing coil voltage should not be used directly to calculate R. The correct ratio is

$$R = \frac{e_2 - n_2}{e_1 - n_1} \quad (11)$$

R, as defined by Equation (10), is dependent upon  $\sigma$  and U through  $e_0$ . R, as defined by Equation (11), eliminates this dependence. Integral circuitry must be incorporated into any instantaneous-reading  $\theta$ -meter for the subtraction of the null signal.

### III. DESCRIPTION OF TRANSDUCERS AND TEST APPARATUS

The  $\theta$ -transducer with two sets of coils mounted on E-laminations is shown in Figure 2. The coils on the outer legs of the E are primary coils, driven at 400 cps. A provision for varying  $\alpha$  was incorporated into the apparatus.

The pancake-coil arrangement, shown in Figure 3, has crossed sensing coils which are attached to the primary coil at a fixed value of  $\alpha = 45$  deg.

A spinning graphite disc was used to test the performance of the transducers, as illustrated in Figure 6. The plastic ring was precision machined on a milling machine index head so that angles could be accurately measured. Signals  $e_1$  and  $e_2$  were measured as a function of  $\theta$ . The graphite disc, which has a 36-inch diameter, was run at 100 rpm.



Figure 6. Transducer Mounted above Graphite Disc

#### IV. EXPERIMENTAL RESULTS

##### A. E-LAMINATION TRANSDUCER

Experiments were conducted to determine the relationship of the flow angle,  $\theta$ , and signal,  $R$ . Results of the experiments conducted for both positive and negative  $\theta$ , with  $\alpha$  equal to 30 deg and 45 deg, are plotted in Figures 4 and 5. Curves based on calculated data are included in the plots for comparison with experimental data.

One prediction of the analysis is that the signal should vary in direct proportion to the cosine of the angle between the flow and transducer.

Observed signals for  $\alpha = 30$  deg and for the cosine of  $\alpha \pm \theta$  have been plotted in Figure 7. A similar graph for  $\alpha = 45$  deg is shown in Figure 8. The signals appear to vary directly with the cosine, within the experimental error, as predicted.

For positive  $\theta$ , the observed signal ratio ( $R$ ) seems to be consistently low. Reference to Figure 7 shows that  $e_2/e_{20}$  is less than the cosine of  $\alpha + \theta$ . If  $e_{20}$ , that is, the value of  $e_2$  when  $\alpha + \theta = 0$ , were changed from the observed value to a smaller magnitude, then  $e_2/e_{20}$  would be an improved fit to the cosine curve.

For negative  $\theta$ , the experimental data (obtained with  $\alpha$  set at 30 deg) agree with the calculated values. For  $\alpha = 45$  deg, the measured angle is less than the actual angle. Reference to Figure 8 shows that the value of  $e_1/e_{10}$  exceeds the cosine of  $\alpha - \theta$ ; this causes the experimental data points to fall below the calculated curve.

Corrections were made for the fact that the coil set closest to the disc axis was exposed to a lower velocity.



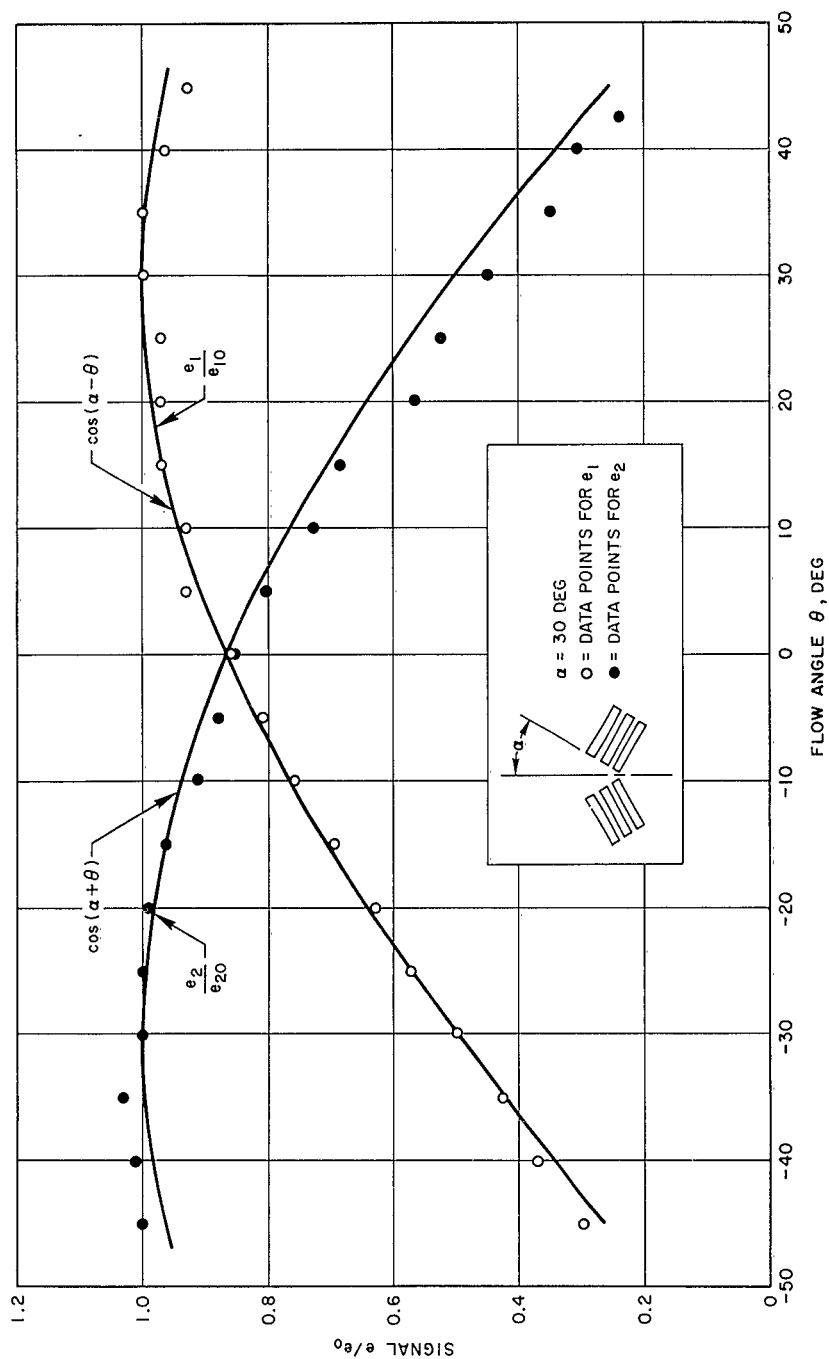


Figure 7. Plots of Observed Signals  $e_1$  and  $e_2$  and of  $\cos(\alpha \pm \theta)$  for  $\alpha = 30^\circ$

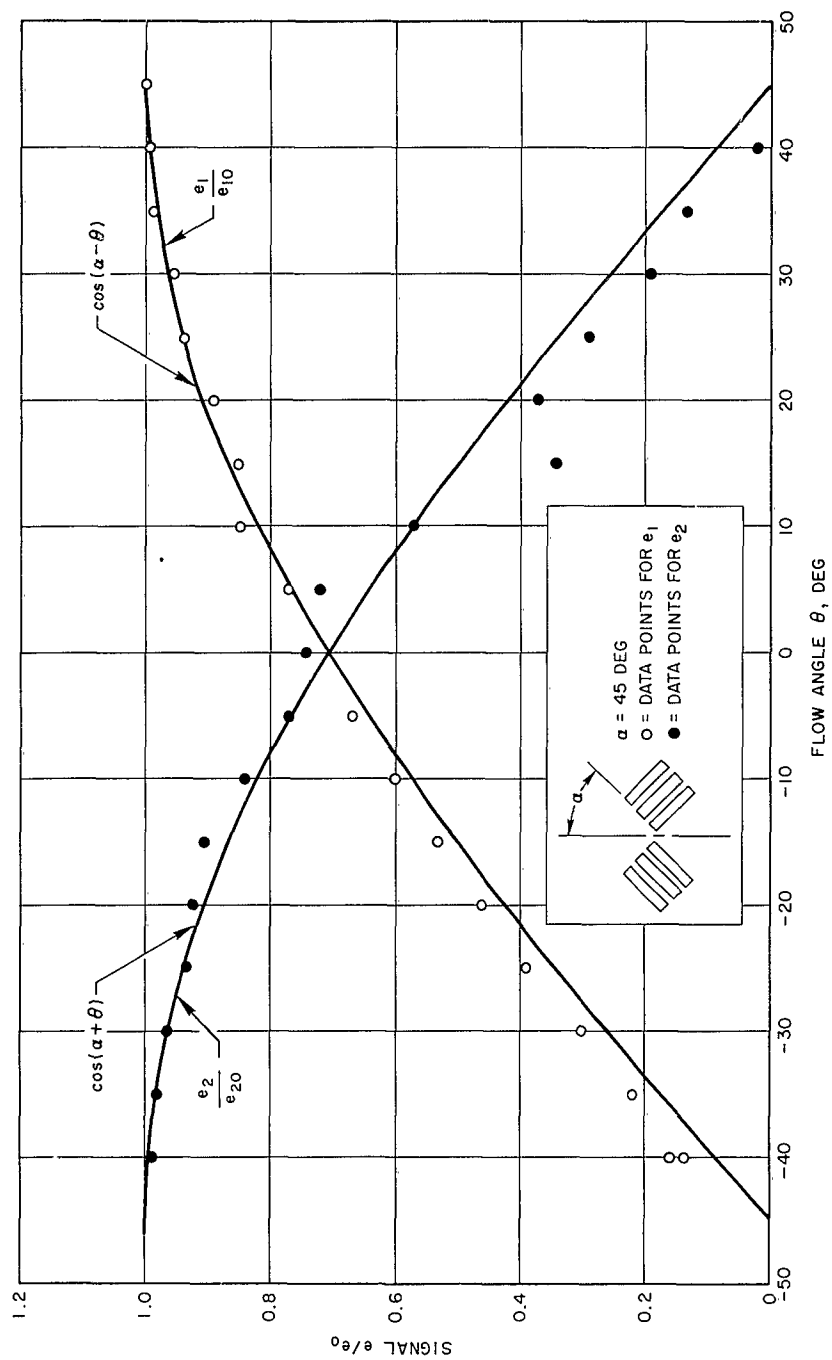


Figure 8. Plots of Observed Signals  $e_1$  and  $e_2$  and of  $\cos(\alpha \pm \theta)$  for  $\alpha = 45^\circ$

## B. PANCAKE COIL GEOMETRY

Measurements of positive  $\theta$  were obtained at five distinct settings of  $\theta$ , varied by 10 degrees between 0 and 40 deg. The results are shown in Figure 9.

Experimental data points are depicted by heavy dots. The solid curve represents  $R$  as a function of  $\theta$ , calculated from Equation (8). The broken curves represent calculated values which have been adjusted by adding or subtracting 1 deg + 5%, as indicated on the drawing. Note that most of the experimental data points fall within the broken curves. This provides an indication of the precision of measurement in the experiment.

## V. CONCLUSIONS

Two different coil arrangements can be used to measure the flow angle.

The signal level varies directly with the cosine of the flow angle. Thus, Equation (8) represents a valid calibration formula for the transducers. The precision of measurement is  $\pm (1 \text{ deg} + 5\%)$ .

Tests were conducted using a graphite disc. The transducers have not been tested with a plasma flow.

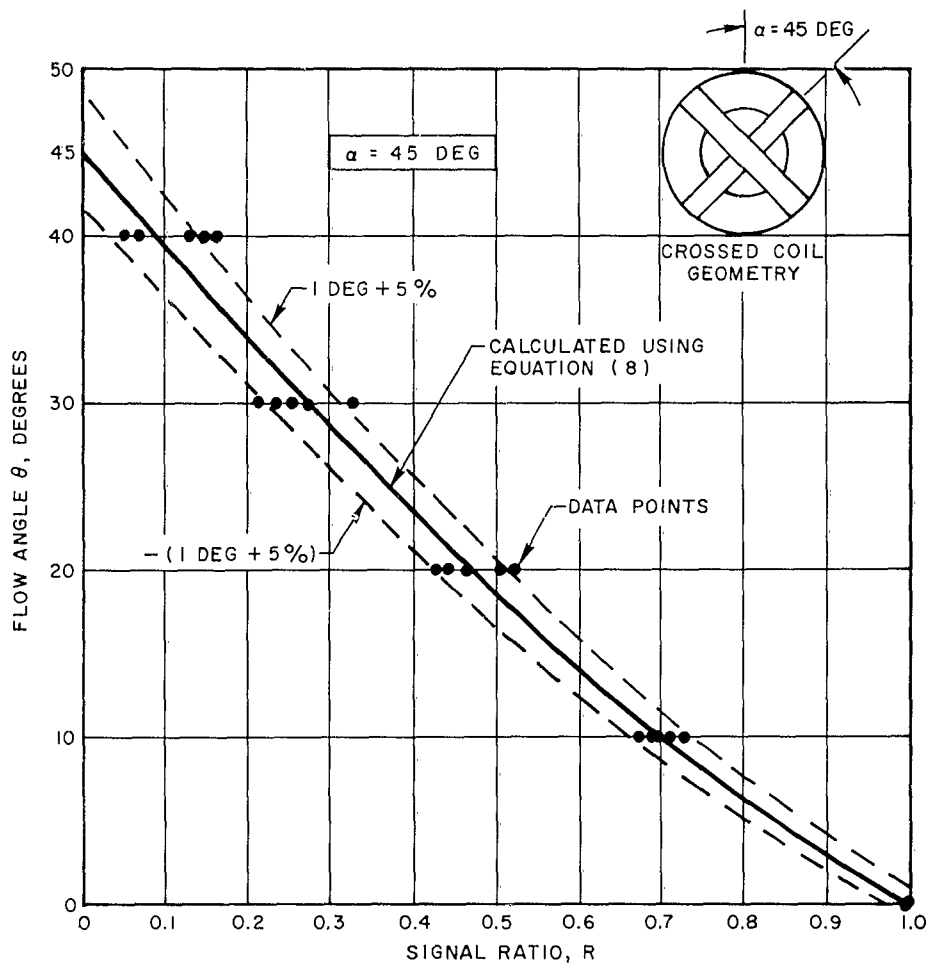


Figure 9. Comparison of Calculated and Experimentally-Determined Flow Angle as a Function of Signal Ratio

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